

CDA 3103: Study Set 1

BINARY CONVERSIONS, SIGNED BINARY VALUES, BINARY ADDITION,
BINARY SUBTRACTION, OVERFLOW

Review: Converting from Binary (Unsigned)

Given a value in binary, separate each bit from most significant to least significant and apply the following formula. The least significant bit is a_0 , the next least significant bit is a_1 , and so on. “b” is the base of the original number and in this case should be 2.

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_1 b + a_0$$

Example: Converting from Binary (Unsigned)

Given: Let $A = (1100\ 1111)_2$. What decimal value does this represent if A is unsigned?

**Partial
Credit 1:**

Place the bits of A into the equation with b as the base. “ k ” is zero for the least significant bit and increases by 1 until it reaches the most significant bit.

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_1 b + a_0$$

Solution 1:

$$n = 1*b^7 + 1*b^6 + 0*b^5 + 0*b^4 + 1*b^3 + 1*b^2 + 1*b + 1$$

$$n = 128 + 64 + 0 + 0 + 8 + 4 + 2 + 1$$

$$N = 207$$

Review: Converting to Binary (Unsigned)

Given a decimal value, we can convert it to binary by repeatedly performing integer division with 2 as the denominator. At each step we keep the remainder of that division operator. We divide until the quotient is 0. Integer division can be represented with the following formula; where n is the decimal value, b is the desired base (2), q is the quotient and a is the remainder.

$$n = bq_0 + a_0$$

Example: Converting to Binary (Unsigned)

Given: Convert $(55)_{10}$ to binary.

Partial Credit 1: Use the division formula to divide 55 by 2 using integer division. Take the quotient and divide it by 2 using integer division. Continue to divide the quotient by 2 until it becomes 0. The remainders will form the binary expansion: the first remainder is the least significant bit and the final remainder is the most significant bit.

$$55 = 2 \cdot 27 + 1 \quad \leftarrow \text{the remainder here is the least significant bit}$$

$$27 = 2 \cdot 13 + 1$$

$$13 = 2 \cdot 6 + 1$$

Solution 1:

$$6 = 2 \cdot 3 + 0$$

$$3 = 2 \cdot 1 + 1$$

$$1 = 2 \cdot 0 + 1 \quad \leftarrow \text{continue until the quotient becomes 0, the remainder here is the most significant bit}$$

$$(55)_{10} = (110111)_2$$

Example: Converting to Binary (Unsigned)

Given: Convert $(55)_{10}$ to binary. Show your answer using 8 bits.

Partial Credit 1: Some questions will ask you to show your answer using a certain number of bits. When values are unsigned, you can pad your result with leading zeroes that do not affect the value. We calculated the value of 55 on the previous slide.

Solution 1: $(55)_{10} = (0011\ 0111)_2$

Given: Convert $(55)_{10}$ to binary. Show your answer using 16 bits.

Partial Credit 1: When values are unsigned, you can pad your result with leading zeroes that do not affect the value. Regardless of the number of zeroes in front, the value of 55 remains the same.

Solution 1: $(55)_{10} = (0000\ 0000\ 0011\ 0111)_2$

Review: Signed Representations

To represent negative numbers we dedicate one of our bits to be the sign bit.

The sign bit is the most significant bit (MSB)

- 1 represents a negative number
- 0 represents a positive number

Signed binary number can still be positive and zero. Now they can also be negative.

Review: Sign-Magnitude

In the Sign-Magnitude representation, the first bit represents the sign and the rest is the magnitude (also known as the absolute value or distance from zero).

Example: Converting from Binary (Sign-Magnitude)

Given: Let $A = (1100\ 1111)_2$. What decimal value does this represent if A is represented with the Sign-Magnitude representation?

Partial Credit 1: Place all of the bits of A **except the most significant** into the conversion equation with b as the base. “ k ” is zero for the least significant bit and increases by 1 until it reaches the most significant bit. If the MSB is 0, there is no need to modify the result. If the MSB is 1, multiply the result by -1.

Solution 1: $n = a_{k-1}b^{k-1} + \dots + a_1b + a_0$ <- leave out the most significant bit (MSB)

$$n = 1*b^6 + 0*b^5 + 0*b^4 + 1*b^3 + 1*b^2 + 1*b + 1$$

$$n = 64 + 0 + 0 + 8 + 4 + 2 + 1$$

$$n = 79$$

<- calculate the magnitude

$$n = -79$$

<- since the MSB is 1, multiply the magnitude by -1

Example: Converting to Binary (Sign-Magnitude)

Given: Convert $(49)_{10}$ to binary using the sign-magnitude representation of signed integers. Show your answer using 8 bits.

Convert 49 to binary and expand to 8 bits. Since this number is positive, ensure the MSB is 0.

**Partial
Credit 1:**

$$49 = 2 \cdot 24 + 1$$

$$24 = 2 \cdot 12 + 0$$

Solution 1: $12 = 2 \cdot 6 + 0$

$$6 = 2 \cdot 3 + 0$$

$$3 = 2 \cdot 1 + 1$$

$$1 = 2 \cdot 0 + 1$$

$$(49)_{10} = (110001)_2 = (0011\ 0001)_2$$

Example: Converting to Binary (Sign-Magnitude)

Given: Convert $(-55)_{10}$ to binary using the sign-magnitude representation of signed integers. Show your answer using 8 bits.

Partial Credit 1: First, calculate the magnitude of -55.

$$55 = 2 * 27 + 1$$

$$27 = 2 * 13 + 1$$

Solution 1: $13 = 2 * 6 + 1$

$$6 = 2 * 3 + 0$$

$$3 = 2 * 1 + 1$$

$$1 = 2 * 0 + 1$$

$$(55)_{10} = (110111)_2$$

Example: Converting to Binary (Sign-Magnitude)

Given: Convert $(-55)_{10}$ to binary using the sign-magnitude representation of signed integers. Show your answer using 8 bits.

Partial Credit 2: Second, expand the magnitude of -55 to 8 bits.

$$(55)_{10} = (110111)_2$$

Solution 2: $(55)_{10} = (0011\ 0111)_2$

Partial Credit 3: Finally, since the original value is negative, set the sign bit to 1.

Solution 3: $(-55)_{10} = (1011\ 0111)_2$

Review: 1's Complement

In the 1's Complement representation every negative number is the bitwise inverse of its positive counterpart. The first bit is the sign bit and indicates whether the number is positive or negative. A 1 means the number is negative – perform a bitwise inverse to see the magnitude of the number. A 0 in the most significant position means the number is positive or zero. No modifications need to be made.

Example: Converting from Binary (1's Complement)

Given: Let $A = (0100\ 1111)_2$. What decimal value does this represent if A is represented with the 1's Complement representation?

Partial Credit 1: Examine the most significant bit. Since it is 0, this number is positive. Place the bits into the conversion equation with b as the base. “ k ” is zero for the least significant bit and increases by 1 until it reaches the most significant bit. Since the MSB is 0 there is no need to modify the result.

Solution 1:

$$n = a_{k-1}b^{k-1} + \dots + a_1b + a_0 \quad \leftarrow \text{the most significant bit (MSB) is zero}$$

$$n = 1*b^6 + 0*b^5 + 0*b^4 + 1*b^3 + 1*b^2 + 1*b + 1$$

$$n = 64 + 0 + 0 + 8 + 4 + 2 + 1$$

$$n = 79 \quad \leftarrow \text{calculate the value}$$

Example: Converting from Binary (1's Complement)

Given: Let $A = (1100\ 1111)_2$. What decimal value does this represent if A is represented with the 1's Complement representation?

Partial Credit 1: Examine the most significant bit. Since it is 1, this number is negative. Perform a bitwise inverse.

Solution 1: 0011 0000

Partial Credit 2: Place the inverted bits into the conversion equation with b as the base. "k" is zero for the least significant bit and increases by 1 until it reaches the most significant bit. Since the **original** MSB was 1, multiply the result by -1.

$$n = a_{k-1}b^{k-1} + \dots + a_1b + a_0$$

$$n = 0*b^7 + 0*b^6 + 1*b^5 + 1*b^4 + 0*b^3 + 0*b^2 + 0*b + 0$$

Solution 2: $n = 32 + 16 + 0 + 0 + 0 + 0$

$$n = 48$$

<- calculate the result

$$n = -48$$

<- since the original MSB was 1, multiply the result by -1

Example: Converting to Binary (1's Complement)

Given: Convert $(49)_{10}$ to binary using the 1's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 1: Convert 49 to binary and expand to 8 bits. Since this number is positive, no modifications are needed.

$$49 = 2 * 24 + 1$$

$$24 = 2 * 12 + 0$$

Solution 1: $12 = 2 * 6 + 0$

$$6 = 2 * 3 + 0$$

$$3 = 2 * 1 + 1$$

$$1 = 2 * 0 + 1$$

$$(49)_{10} = (110001)_2 = (0011\ 0001)_2$$

Example: Converting to Binary (1's Complement)

Given: Convert $(-55)_{10}$ to binary using the 1's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 1: First, calculate the positive form of -55.

$$55 = 2 * 27 + 1$$

$$27 = 2 * 13 + 1$$

Solution 1: $13 = 2 * 6 + 1$

$$6 = 2 * 3 + 0$$

$$3 = 2 * 1 + 1$$

$$1 = 2 * 0 + 1$$

$$(55)_{10} = (110111)_2$$

Example: Converting to Binary (1's Complement)

Given: Convert $(-55)_{10}$ to binary using the 1's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 2: Second, expand the positive form of -55 to 8 bits.

$$(55)_{10} = (110111)_2$$

Solution 2: $(55)_{10} = (0011\ 0111)_2$

Partial Credit 3: Finally, since the original value is negative, perform a bitwise inverse.

Solution 3: $(-55)_{10} = (1100\ 1000)_2$

Review: 2's Complement

2's Complement is the standard representation for signed integers. To convert between positive and negative values, we calculate a bitwise inverse and add 1 to the result. The first bit is the sign bit and indicates whether the number is positive or negative. A 1 means the number is negative. A 0 in the most significant position means the number is positive or zero.

Example: Converting from Binary (2's Complement)

Given: Let $A = (0100\ 1111)_2$. What decimal value does this represent if A is represented with the 2's Complement representation?

Partial Credit 1: Examine the most significant bit. Since it is 0, this number is positive. Place the bits into the conversion equation with b as the base. "k" is zero for the least significant bit and increases by 1 until it reaches the most significant bit. Since the MSB is 0 there is no need to modify the result.

Solution 1:

$$n = a_{k-1}b^{k-1} + \dots + a_1b + a_0$$

<- the most significant bit (MSB) is zero

$$n = 1*b^6 + 0*b^5 + 0*b^4 + 1*b^3 + 1*b^2 + 1*b + 1$$

$$n = 64 + 0 + 0 + 8 + 4 + 2 + 1$$

$$n = 79$$

<- calculate the value

Example: Converting from Binary (2's Complement)

Given: Let $A = (1100\ 1111)_2$. What decimal value does this represent if A is represented with the 2's Complement representation?

Partial Credit 1: Examine the most significant bit. Since it is 1, this number is negative. Perform a bitwise inverse and add 1.

Solution 1: 0011 0001

Partial Credit 2: Place the modified bits into the conversion equation with b as the base. "k" is zero for the least significant bit and increases by 1 until it reaches the most significant bit. Since the **original** MSB was 1, multiply the result by -1.

$$n = a_{k-1}b^{k-1} + \dots + a_1b + a_0$$

$$n = 0*b^7 + 0*b^6 + 1*b^5 + 1*b^4 + 0*b^3 + 0*b^2 + 0*b + 1$$

Solution 2: $n = 32 + 16 + 0 + 0 + 0 + 1$

$$n = 49$$

<- calculate the result

$$n = -49$$

<- since the original MSB was 1, multiply the result by -1

Example: Converting to Binary (2's Complement)

Given: Convert $(49)_{10}$ to binary using the 2's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 1: Convert 49 to binary and expand to 8 bits. Since this number is positive, no modifications are needed.

$$49 = 2 * 24 + 1$$

$$24 = 2 * 12 + 0$$

Solution 1: $12 = 2 * 6 + 0$

$$6 = 2 * 3 + 0$$

$$3 = 2 * 1 + 1$$

$$1 = 2 * 0 + 1$$

$$(49)_{10} = (110001)_2 = (0011\ 0001)_2$$

Example: Converting to Binary (2's Complement)

Given: Convert $(-55)_{10}$ to binary using the 2's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 1: First, calculate the positive form of -55.

$$55 = 2 * 27 + 1$$

$$27 = 2 * 13 + 1$$

Solution 1: $13 = 2 * 6 + 1$

$$6 = 2 * 3 + 0$$

$$3 = 2 * 1 + 1$$

$$1 = 2 * 0 + 1$$

$$(55)_{10} = (110111)_2$$

Example: Converting to Binary (2's Complement)

Given: Convert $(-55)_{10}$ to binary using the 2's complement representation of signed integers. Show your answer using 8 bits.

Partial Credit 2: Second, expand the positive form of -55 to 8 bits.

$$(55)_{10} = (110111)_2$$

Solution 2: $(55)_{10} = (0011\ 0111)_2$

Partial Credit 3: Finally, since the original value is negative, perform a bitwise inverse and add 1.

Solution 3: $(-55)_{10} = (1100\ 1001)_2$