

CDA 3103: Study Set 7

PERFORMANCE EQUATIONS, ANALYSIS OF PERFORMANCE

Review: Performance Equations

Most of our performance calculations are based on the CPU performance equation:

$$\text{CPU Time} = \text{Instruction Count} \times \text{CPI} \times \text{Clock Cycle}$$

Which uses the number of instructions for our program, the number of cycles per instruction, and the clock cycle **time** to determine the amount of time our program will spend in the CPU.

The Clock Cycle time in this equation is the clock cycle time. It is often a very small number. This value is also the inverse of the **clock cycle rate** – the number of cycles that can be complete in a second – which is often a very large number.

Review: Performance Equations

In addition to primary CPU performance equation, we looked at a few relations that helped form that equation.

$$\text{CPU Time} = \text{CPU Clock Cycles} \times \text{Clock Cycle Time}$$

The amount of time a program will spend in the CPU is determined by the number of cycles it will spend in the CPU and the time it takes to complete each cycle.

$$\text{CPU Clock Cycles} = \text{Instruction Count} \times \text{Cycles Per Instruction}$$

The number of CPU clock cycles can be approximated with the number of instructions and the average number of cycles it takes to complete an instruction.

Review: Performance Equations

The CPI – Cycles Per Instruction – depends on the architecture we have created.

- On a single-cycle datapath each instruction completes in one cycle. The CPI must equal 1.
- On a pipelined datapath each instruction completes in a number of cycles equal to the number of pipeline stages. In the MIPS architecture the CPI would equal 5.
- In some architectures different instructions take differing numbers of clock cycles. In these cases the CPI is an average of all the instruction times:

$$\text{CPI} = \sum_{i=1}^n (P_i \times \text{CPI}_i)$$

Review: Performance Equations

Often we will compare two CPUs; either because we have two separate systems or because we wish to evaluate a change we are making to one system. We can compare the performance of these two CPUs to judge which is faster.

$$\text{Performance}(X) = \frac{1}{\text{Execution Time}(X)}$$

$$\frac{\text{Performance}(X)}{\text{Performance}(Y)} = \frac{\text{Execution Time}(Y)}{\text{Execution Time}(X)} = n$$

Example: CPU Performance Equation

Given: Suppose you wish to run a program that has 4×10^9 instructions on a 3.5 GHz machine with a CPI of 2. What is the expected CPU time? Round your answers to 3 decimal places.

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Solution 1: To determine the expected CPU time we should use the CPU Performance Equation.

Partial Credit 1: CPU Time = Instruction Count * CPI * Clock Cycle Time

Example: CPU Performance Equation

Given: Suppose you wish to run a program that has 4×10^9 instructions on a 3.5 GHz machine with a CPI of 2. What is the expected CPU time? Round your answers to 3 decimal places.

Solution 2: We are given the number of instructions, the CPI, and the clock cycle rate in the problem statement. The clock cycle rate is inversely proportional to the clock cycle time. Gigahertz is 10^9 Hertz. We place these numbers into the equation:

Partial
Credit 2: CPU Time = $4 \times 10^9 * 2 * (1 / 3.5 * 10^9)$

Example: CPU Performance Equation

Given: Suppose you wish to run a program that has 4×10^9 instructions on a 3.5 GHz machine with a CPI of 2. What is the expected CPU time? Round your answers to 3 decimal places.

Solution 3: Now we can reduce to find the CPU time in seconds:

Partial CPU Time = $4 * 2 / 3.5$
Credit 3: CPU Time = 2.286 seconds

Example: Calculation CPI

Given: Suppose you have the following instruction mix. What is the CPI?

Instruction Type	Frequency (%)	Cycles
Memory	25	2
ALU	45	1
Branch	25	2
Jump	5	1

Example: Calculation CPI

Given: Suppose you have the following instruction mix. What is the CPI?

Instruction Type	Frequency (%)	Cycles
Memory	25	2
ALU	45	1
Branch	25	2
Jump	5	1

Solution 1: We are given the frequency of all the instruction types in percentages and the number of cycles each instruction type requires to complete. We can use the summation for CPI to determine the average cycles per instruction. P is the probability for each instruction type and CPI is the cycles needed for that particular instruction type.

Partial Credit 1:
$$CPI = \sum P_i * CPI_i$$

Example: Calculation CPI

Given: Suppose you have the following instruction mix. What is the CPI?

Instruction Type	Frequency (%)	Cycles
Memory	25	2
ALU	45	1
Branch	25	2
Jump	5	1

Solution 2: Since there are four instruction types, we will need to sum the four frequencies multiplied by their individual CPIs.

Partial Credit 2: $\text{CPI} = .25 * 2 + .45 * 1 + .25 * 2 + .05 * 1$

Example: Calculation CPI

Given: Suppose you have the following instruction mix. What is the CPI?

Instruction Type	Frequency (%)	Cycles
Memory	25	2
ALU	45	1
Branch	25	2
Jump	5	1

Solution 3: The result of the summation is the average cycles per instruction for this particular instruction mix. Changing the frequency of instruction types or the number of cycles for a given instruction type would change the result.

**Partial
Credit 3:** CPI = 1.5

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- If the processors each execute a program in 20 seconds, find the number of cycles and the number of instructions for each processor.
 - We are trying to reduce the program execution time by 20%, but this leads to an increase of 25% in the CPI. What clock rate should we have to get this time reduction for each processor?

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- If the processors each execute a program in 20 seconds, find the number of cycles and the number of instructions for each processor.

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 1: This question has several parts. First, we should determine the number of cycles needed for each processor. Since we have the CPU Time and the Clock Rate, we should use this equation:
$$\text{CPU Time} = \text{CPU Clock Cycles} * \text{Clock Rate}$$

**Partial
Credit 1:** $20 = \text{P1Cycles} * (1/3 * 10^9)$
 $\text{P1Cycles} = 20 * 3 * 10^9$
 $\text{P1Cycles} = 60 * 10^9$

Remember that Clock Rate is the number of cycles per second. $1 / \text{Clock Rate}$ will give us the clock cycle time.

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- If the processors each execute a program in 20 seconds, find the number of cycles and the number of instructions for each processor.

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 2: This question has several parts. First, we should determine the number of cycles needed for each processor. P1 has $60 * 10^9$ cycles. Now we should repeat the equation for the second processor.

Partial Credit 2:

$$20 = P2Cycles * (1/4 * 10^9)$$
$$P2Cycles = 20 * 4 * 10^9$$
$$P2Cycles = 80 * 10^9$$

Remember that Clock Rate is the number of cycles per second. $1 / \text{Clock Rate}$ will give us the clock cycle time.

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- If the processors each execute a program in 20 seconds, find the number of cycles and the number of instructions for each processor.

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 3: Now that we have the number of cycles for both processors we can calculate the number of instructions using this equation: $\text{CPU Clock Cycles} = \text{Instruction Count} * \text{CPI}$. Let's start with the first processor:

Partial Credit 3: $\text{CPU Clock Cycles} = \text{Instruction Count} * \text{CPI}$
 $60 * 10^9 = \text{Instruction Count P1} * 1.5$
 $\text{Instruction Count P1} = 40 * 10^9$

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- If the processors each execute a program in 20 seconds, find the number of cycles and the number of instructions for each processor.

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 4: To complete the first part of the question, we need to calculate the number of instructions for the second processor as well:

Partial Credit 4:

$$\text{CPU Clock Cycles} = \text{Instruction Count} * \text{CPI}$$
$$80 * 10^9 = \text{Instruction Count P2} * 2$$
$$\text{Instruction Count P2} = 40 * 10^9$$

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- We are trying to reduce the program execution time by 20%, but this leads to an increase of 25% in the CPI. What clock rate should we have to get this time reduction for each processor?

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 5: The second part of the question asks us to consider the effect of a modification to our processors. The program execution time will be reduced by 20% and the CPI will be increased by 25%. We should calculate new values for these pieces of information.

Partial Credit 5:

$$\text{New Execution Time} = \text{Old Execution Time} * (1 - .2)$$
$$\text{New Execution Time} = 20 * .8$$
$$\text{New Execution Time} = 16.000$$

Example: Analysis of Performance

- Given:** Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.
- We are trying to reduce the program execution time by 20%, but this leads to an increase of 25% in the CPI. What clock rate should we have to get this time reduction for each processor?

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 6: The second part of the question asks us to consider the effect of a modification to our processors. The program execution time will be reduced by 20% and the CPI will be increased by 25%. We should calculate new values for these pieces of information.

**Partial
Credit 6:** New CPI = Old CPI * (1 + .25)
New CPI P1 = 1.5 * 1.25
New CPI P1 = 1.875

New CPI P2 = 2 * 1.25
New CPI P2 = 2.500

Example: Analysis of Performance

Given:

Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.

- We are trying to reduce the program execution time by 20%, but this leads to an increase of 25% in the CPI. What clock rate should we have to get this time reduction for each processor?

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 7: Now that we know the new execution time and the new CPI values, we can calculate the new clock rates. We can use the CPU performance equation: $\text{CPU Time} = \text{Instruction Count} * \text{CPI} * \text{Clock Cycle}$. The number of instructions that will be executed has not changed, so we can use those values from part 1.

**Partial
Credit 7:**

$\text{CPU Time} = \text{Instruction Count} * \text{CPI} * \text{Clock Cycle}$
 $\text{CPU Time} = \text{IC P1} * \text{New CPI P1} * (1 / \text{New Clock Cycle Rate P1})$
 $16 = 40 * 10^9 * 1.875 * (1 / \text{New Clock Cycle Rate P1})$
 $16 * \text{New Clock Cycle Rate P1} = 75 * 10^9$
 $\text{New Clock Cycle Rate P1} = 75 / 16 * 10^9$
 $\text{New Clock Cycle Rate P1} = 4.688 * 10^9$

 $\text{New Clock Cycle Rate P1} = 4.688 \text{ GHz}$

Example: Analysis of Performance

Given:

Consider two different processors P1 and P2 executing the same instruction set with the clock rates and CPIs given in the following table. Round your answers to 3 decimal places.

- We are trying to reduce the program execution time by 20%, but this leads to an increase of 25% in the CPI. What clock rate should we have to get this time reduction for each processor?

Processor	Clock Rate	CPI
P1	3 GHz	1.5
P2	4 GHz	2

Solution 8: Now that we know the new execution time and the new CPI values, we can calculate the new clock rates. We can use the CPU performance equation: $\text{CPU Time} = \text{Instruction Count} * \text{CPI} * \text{Clock Cycle}$. The number of instructions that will be executed has not changed, so we can use those values from part 1.

**Partial
Credit 8:**

$\text{CPU Time} = \text{Instruction Count} * \text{CPI} * \text{Clock Cycle}$
 $\text{CPU Time} = \text{IC P2} * \text{New CPI P2} * (1 / \text{New Clock Cycle Rate P2})$
 $16 = 40 * 10^9 * 2.5 * (1 / \text{New Clock Cycle Rate P2})$
 $16 * \text{New Clock Cycle Rate P2} = 100 * 10^9$
 $\text{New Clock Cycle Rate P2} = 100 / 16 * 10^9$
 $\text{New Clock Cycle Rate P2} = 6.250 * 10^9$

 $\text{New Clock Cycle Rate P2} = 6.250 \text{ GHz}$