

1. Read each of the following descriptions. For each, indicate whether it is describing a *derivative* or an *integral*. Re-write the description as an equation using mathematical notation.

- The probability that a standard Normal variable is between -1 and 1 is the *area* under the curve $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ within that range, which is roughly 0.683 .
- An object's height position at time t is $y = t^3 + 5t - 7$. It's velocity is the function describing how that position is *changing*, in this case $3t^2 + 5$.
- The total revenue for some company given the number of items they are producing (x) is known to be $-x^3 + 450x^2 + 52,500x$. Economists describe the *marginal revenue* as the *change* in total revenue, in this case $-3t^2 + 900t + 52,500$.
- The force between two charged particles is proportional to the product of the charge values and inversely proportional to the square of their distances: $f(r) = \frac{kq_1q_2}{r^2}$, where r is the distance in meters, q_1 and q_2 are the charge of the first and second particle in coulombs, and k is a constant. We can compute the *work* done by the particles movements as a result of their charges by calculating the *area* under the force curve with respect to the distance.

2. Use Wolfram Alpha to compute the following.

- derivative $x^2 - 2x + 3$
- integrate $y^3 - 9$
- integrate $3w^2 + 2w - 9$
- derivative $x \sin(x^2)$

3. Evaluate the following by hand.

- $\frac{d}{dx}(x^2 - 3x + 2)$
- $\frac{d}{dx}(-9x^2 - x)$
- $\frac{d}{dw}(w - 4)$
- $\frac{d}{dx}(-3.41)$

4. Evaluate the following by hand.

- $\int 2x \, dx$
- $\int (-3y^2 + 2y - 1) \, dy$
- $\int_0^1 -w^2 \, dw$
- $\int_{\frac{1}{2}}^3 4 \, dx$

5. Evaluate the following by hand.

- $\lim_{x \rightarrow 2}(x^2 - 4)$
- $\lim_{y \rightarrow 1} \frac{y^2 - 2y + 1}{y^3 - y}$
- $\lim_{w \rightarrow 2} \frac{w+1}{w-1}$
- $\lim_{x \rightarrow \infty} \frac{x^2 - 1}{2x^2 + 1}$

6. What is the *order* of the following derivatives?

(a) $f^4(x) = x^2 - 3x + 2$

(c) $\frac{d}{dx} \left(\frac{d}{dx} x^3 \right)$

(b) $f'''(y) = \sin(y) + 3$

(d) $\ddot{y} = -gt$

7. Evaluate the following by hand.

(a) $\frac{d}{dz} (3z^3 + 2z^2 - 3z - 1)$

(d) $\frac{d}{dy} (y^5 - 2y)$

(b) $\frac{d}{dx} x^4$

(e) $\frac{d}{dr} (2r^3 + 9)$

(c) $\frac{d}{dw} (w^2 - 2w + 4)$

(f) $\frac{d}{dy} \frac{1}{y}$

8. Evaluate the following by hand.

(a) $\int z^2 dz$

(d) $\int \frac{1}{w} dw$

(b) $\int_1^5 (2y^3 + y^2 + 1) dy$

(e) $\int_{-1}^1 (x^4 - 2x) dx$

(c) $\int_1^5 (-x^4 - x^2 - 1) dx$

(f) $\int \left(\frac{1}{3}w^2 + \frac{1}{2}w \right) dw$

9. Identify the type and order of the following differential equations.

(a) $\dot{x} + x = t^2$

(c) $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} = 0$

(b) $y'' - 2y' + y = 0$

(d) $\frac{du}{dx} = 2u + x^2$

10. The the initial value problem for the following ODEs:

(a) $\frac{dy}{dx} = 10 - x, \quad y(0) = -1$

(b) $y' = 6t^2, \quad y(1) = 5$

(c) $\dot{w} = y^2 - 3y + 1, \quad w(0) = 10$

11. Sketch the directional field for the following ODE, including isoclines and at least one integral curve.

$$y' = x - y$$

12. Objects accelerate downwards due to gravity at $-9.8 \text{ meters/second}^2$. Acceleration on an object is simply it's *change* in velocity, and velocity is simply it's *change* in position. Let v_0 be the initial velocity of the object and s_0 be the initial height of the object. Knowing gravity is acceleration allows us to write a very simple ODE for motion: $s''(t) = -9.8$, $s'(0) = v_0$, $s(0) = s_0$. Solve this to find the general equation for position given time. *Hint: You will need to do this in two stages, first to get velocity then to get position.*